LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - **STATISTICS**

FIRST SEMESTER – **NOVEMBER 2013**

ST 1820 - ADVANCED DISTRIBUTION THEORY

Date : 05/11/2013 Dept. No. Max. : 100 Marks
Time : 1:00 - 4:00

SECTION - A

Answer ALL questions. Each carries TWO marks. (10 x 2 = 20 marks)

1. Define distribution function of a random variable and state its properties.
2. Write the pdf of truncated binomial, left truncated at '0' and find its mean.
3. Show that geometric distribution satisfies lack of memory property.
4. Find the pgf of a log-series distribution. Hence obtain its mgf.
5. Show that the marginals of a bivariate discrete uniform need not be discrete uniform.
6. Let $(X_1, X_2) \sim BVP(\lambda_1, \lambda_2, \lambda_{12})$. Find its pgf.
7. Show that geometric mean of independent log-normal variables is log-normal.
8. If $X_1,\ldots, X_n$ are iid exponential random variables with the parameter α, then find the distribution of $\sum_{i=1}^{n} X_i$.
9. Let $X \sim IG(\mu, \lambda)$. Find the distribution of aX, where $a > 0$.
10. Define non-central chi-square distribution.

SECTION – B

Answer any FIVE questions. Each carries EIGHT marks. (5 x 8 = 40 marks)

11. Let distribution function of X be

$$F(x) = \begin{cases} 0, & x < -1 \\ \frac{(x+2)}{4}, & -1 \leq x < 1 \\ 1, & 1 \leq x < \infty \end{cases}$$

Find the decomposition of F. Hence obtain the pdf of X.

12. State and prove a characterization of Bernoulli distribution through moments.
13. Let X have a power-series distribution. Find the pgf and mgf of X. Hence find its mean.
14. Let $P(s_1, s_2)$ denote the pgf of (X_1, X_2) at (s_1, s_2). Obtain the pgf of the conditional distribution of X_1 given $X_2 = x_2$ at s_1.
15. Let $(X_1, X_2) \sim BVP(\lambda_1, \lambda_2, \lambda_{12})$. Prove that $X_1 \perp\!\!\!\perp X_2 \Leftarrow \lambda_{12} = 0$.
16. State and prove Skitovitch theorem regarding independent normal variables.
17. State and prove characterization of exponential distribution through failure rate function.
18. Let $(X_1, X_2) \sim BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. Find the conditional distribution of $X_2 \mid X_1 = x_1$.

SECTION – C

Answer any TWO questions. Each carries TWENTY marks. (2 x 20 = 40 marks)

19(a). Let X_1 and X_2 be iid geometric random variables. Find the conditional distribution of $X_1 \mid X_1 + X_2 = n$. (8)

(b). For a log-series distribution, obtain the recurrence relation for (i) raw moment μ'_{r+1} and (ii) central moment μ_{r+1}. (12)

20(a). Let $(X_1, X_2) \sim BVP(\lambda_1, \lambda_2, \lambda_{12})$. Find the two regression equations and hence obtain the correlation coefficient between X_1 and X_2. (10)

(b). Let $(X_1, X_2) \sim BB(n, p_1, p_2, p_{12})$. Stating the conditions, Show that (X_1, X_2) tends to $BVP(\lambda_1, \lambda_2, \lambda_{12})$. (10)

21(a). Let X_1, X_2, X_3 be independent normal variables such that $E(X_1) = 1$, $E(X_2) = 3$, $E(X_3) = 2$ and $V(X_1) = 2$, $V(X_2) = 2$, $V(X_3) = 3$. Verify the independence of the following pairs:

(i) $X_1 + X_2$ and $X_1 - X_2$

(ii) $X_1 + X_2 - 2X_3$ and $X_1 - X_2 + 2X_3$

(iii) $2X_1 + X_3$ and $X_2 - X_3$. (10)

(b). Derive the mgf of inverse Gaussian distribution. (10)

22(a). Derive the pdf of non-central t distribution. (16)

(b). Let $X \sim N(\theta, 1)$, θ □ R. Assume that $\theta \sim N(0, 1)$. Find the compound distribution of X. (4)

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